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# **PREDICTION OF GLOBAL AND LOCALIZED DAMAGE AND FUTURE RELIABILITY FOR RC STRUCTURES SUBJECT TO EARTHQUAKES**

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## **SUMMARY**

The paper deals with the prediction of global and localized damage and the future reliability estimation of partly damaged reinforced concrete (RC) structures under seismic excitation. Initially, a global maximum softening damage indicator is considered based on the variation of the eigenfrequency of the first mode due to the stiffness and strength deterioration of the structure. The hysteresis of the first mode is modelled by a Clough and Johnston hysteretic oscillator <sup>1</sup> with a degrading elastic fraction of the restoring force. The linear parameters of the model are assumed to be known, measured before the arrival of the first earthquake from non-destructive vibration tests or via structural analysis. The previous excitation and displacement response time series is employed for the identification of the instantaneous softening using an ARMA model. The hysteresis parameters are updated after each earthquake. The proposed model is next generalized for the MDOF system. Using the adapted models for the structure and the global damage state, the global damage in a future earthquake can then be estimated when a suitable earthquake model is applied. The performance of the model is illustrated on RC frames which were tested by Sözen and his associates <sup>2,4</sup>.

## **1. INTRODUCTION**

The physical local damage in reinforced concrete (RC) structures subject to severe seismic excitation is attributed to micro-cracking and crushing of concrete, yielding of the reinforcement bars and bond deterioration at the steel-concrete interfaces. To the extent



that RC structures can be modelled by non-linear mechanical theories, local damage at a cross-section of the structure can adequately be measured by the degradation of bending stiffness and moment capacity of the cross-section. The overall effect of local damages is the stiffness and strength deterioration of the structure. A global damage indicator can then be defined as a functional of such continuously distributed local damages which characterize the overall damage state and serviceability of the structure.

Global damage indicators are response quantities characterizing the damage state of the structure after an earthquake excitation, and such can be used in decision-making during the design phase, or in case of post-earthquake reliability and repair problems. In serving these purposes, the global damage indicator should, at least, be observable for practical purposes, be a non-decreasing function of time unless the structures are repaired or strengthened, possess a failure surface (serviceability or ultimate limit state) to separate safe states from the unsafe ones and carry Markov property so that post-earthquake reliability estimates for a partly damaged structure can be made solely from the latest recorded value of the damage indicator.

The maximum softening damage indicators measure the maximum relative reduction of the vibrational frequencies for an equivalent linear system with slowly varying stiffness properties during a seismic event, hence, display the combined damaging effects of the maximum displacement ductility of the structure during extreme plastic deformations and the stiffness deterioration in the elastic regime, the latter effect being referred to as final softening. The introduction of the one-dimensional maximum softening indicator based on an equivalent linear single-degree-of-freedom (SDOF) system fits to the first mode of the RC building as a global damage indicator is due to DiPasquale and Çakmak<sup>3</sup>. The excitation and displacement response time series of a single position on the building are the only required observations for the one-dimensional maximum softening damage indicator. The applicability of the index was analysed based on data from shake table experiments with RC frames performed by Sözen and his associates<sup>2,4</sup>. Limit states for slight damage to total collapse were calibrated using this data and the performance of the index was tested for partly damaged structures which had been instrumented in the past. The maximum softening concept has also been generalized to multi-degree-of-freedom (MDOF) models along with the associated damage localization problem<sup>9</sup>. The Markov property of the maximum damage indicator chains for SDOF and 2 DOF models was tested numerically by means of Monte Carlo simulations<sup>8,9</sup> and it was concluded that the global damage indicator fulfils Markov property with sufficient accuracy.

The present paper deals with the prediction of global and localized damage and the future reliability estimation of partly damaged reinforced concrete (RC) structures under seismic excitation. Initially, a global maximum softening damage indicator is considered based on the variation of the eigenfrequency of the first mode due to the stiffness and strength deterioration of the structure. The hysteresis of the first mode is modelled by a Clough and Johnston hysteretic oscillator<sup>1</sup>, with degrading elastic fraction of the restoring force, subject to seismic excitation. The circular eigenfrequency, damping ratio and the modal participation factor of the first mode of the undamaged structure are assumed to be known, measured before the arrival of the first earthquake from

non-destructive vibration tests or via structural analysis. The previous excitation and displacement response time series is employed for the identification of the instantaneous softening using an ARMA model. The two free hysteresis parameters are updated after each earthquake from another system identification procedure where a weighted error criterion defined using instantaneous softening and the displacement response time series are employed. Using the adapted models for the structure and the global damage state, the global damage in a future earthquake can then be estimated if a suitable earthquake model is applied. In the paper, the Markov property of the global maximum softening damage indicator is verified. Hence, the maximum softening index as calculated from the present method may be used for future reliability estimates as well.

The proposed model is next generalized for an MDOF system. The horizontal displacement of the structure is assumed to be measured in a finite  $n$  number of points on the structure. An equivalent hysteretic shear model of  $n$  degrees of freedom is introduced in which the linear system parameters are identified to provide the same undamped circular eigenfrequencies, modal damping ratios and modal participation factors as measured on the undamaged structure. The shear force between the measure points (typically storeys) is next modelled by Clough-Johnston hysteretic models similar to the one applied in the SDOF case. The set of maximum softening damage indices between all measure points forms a Markov vector and can be used for reliability estimation. Since the local damages are directly observed, the localization of the damage problem (the inverse problem) is circumvented.

The performance of the model is illustrated on RC frames which were tested by Sözen and his associates <sup>2,4</sup>.

## 2. HYSTERETIC MODEL FOR SDOF OSCILLATOR

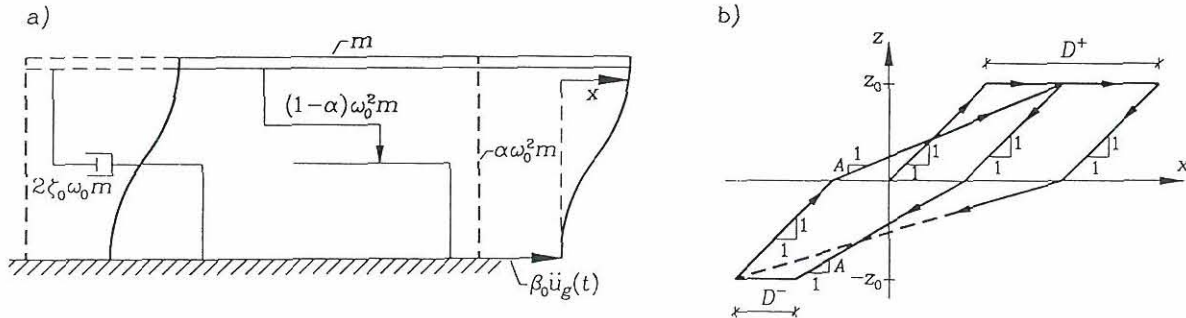


Figure 1. a) SDOF hysteretic oscillator model b) Clough-Johnston hysteretic model.

The equations of motion of the first mode are modelled by the following coupled differential equations

$$\begin{aligned} \ddot{x}(t) + 2\zeta_0\omega_0\dot{x}(t) + \omega_0^2 \left[ \alpha(D(t))x(t) + (1 - \alpha(D(t)))z(t) \right] = \\ -\beta_0\ddot{u}_g(t) \quad , \quad t > t_0 \quad , \quad x(t_0) = \dot{x}(t_0) = 0 \end{aligned} \quad (1)$$



$$\dot{z}(t) = k(\dot{x}(t), z(t), D(t); z_0) \dot{x}(t) \quad , \quad z(t_0) = 0 \quad (2)$$

$$\dot{D}(t) = g(\dot{x}(t), z(t); z_0) \dot{x}(t) \quad , \quad D(t_0) = D_0 \quad (3)$$

$$\alpha(D(t)) = \left( \frac{2z_0}{2z_0 + D(t)} \right)^{n_0} \quad (4)$$

$$k(\dot{x}(t), z(t), D(t); z_0) = H(z) \{ A(t) H(\dot{x}) (1 - H(z - z_0)) + H(-\dot{x}) \} + H(-z) \{ A(t) H(-\dot{x}) (1 - H(-z - z_0)) + H(\dot{x}) \} \quad (5)$$

$$g(\dot{x}(t), z(t); z_0) = H(\dot{x}) H(z - z_0) - H(-\dot{x}) H(-z - z_0) \quad (6)$$

$$A(t) = \frac{z_0}{z_0 + D(t)} \quad (7)$$

$$H(x) = \begin{cases} 1 & , \quad x \geq 0 \\ 0 & , \quad x < 0 \end{cases} \quad (8)$$

The first modal coordinate  $x(t)$  can be defined as the top storey displacement of the structure relative to the ground surface if the mode shape is suitably normalized. The linear circular eigenfrequency,  $\omega_0$ , the damping ratio,  $\zeta_0$ , and the mode participation factor,  $\beta_0$ , of the first mode are assumed to be known before the arrival of the first earthquake, obtained previously via linear structural analysis or non-destructive experimentation of the structure.  $\ddot{u}_g(t)$  indicates the horizontal earth surface acceleration signal and the first earthquake starts at the time  $t = t_0$ .  $\alpha(D(t))$  is the elastic fraction of the restoring force.

$z(t)$  is the hysteretic component which is modelled using the Clough-Johnston hysteretic model.  $z = +z_0$ ,  $z = -z_0$  signify the yield levels.  $k(\dot{x}(t), z(t), D(t); z_0)$  is a non-analytic function describing the state dependent stiffness of the hysteretic model on the component  $z(t)$ . The stiffness degrading hysteretic constitutive law of the model can be represented as shown in figure 1.b. The Clough-Johnston model deals with the stiffness degradation by changing the slope  $A(t)$  of the elastic branches as the accumulated plastic deformations,  $D^+(t)$  and  $D^-(t)$  at positive and negative yielding, increase as shown in figure 1.b.  $D(t) = D^+(t) + D^-(t)$  is the total accumulated plastic deformations. For loading branches, the slope  $A(t)$  is selected such that the elastic branch always

aims at the previous unloading point with the other sign. At unloadings, the slope is 1.  $D_0$  is the initial value of the total accumulated damage which is zero before the first earthquake hits and is assumed to be determined from previous earthquake and displacement response records for the succeeding earthquakes.  $H(x)$  is the unit step function.

A novelty of the present model stems primarily from the modelling of  $\alpha(D(t))$  as a non-increasing function of the damage parameter  $D(t)$ . Since,  $\alpha(D(t))$  measures the fraction of the restoring force from linear elastic behaviour, this fraction must decrease as larger and larger parts of the structure become plastic. Note that initially,  $\alpha(D(0)) = 1$ , and, unless there is damage, still  $\alpha(0) = 1$ . The dependence of  $\alpha(D(t))$  on  $D(t)$  as indicated by (4) has been selected to fulfil this boundary condition. The relative success of the model (1)-(8) in reproducing actually recorded displacement time series in the studied example is primarily due to this modelling.

The hysteretic parameters  $z_0, n_0$  are to be identified from the experienced excitation and displacement response time series with an optimal system identification method. The Clough-Johnston hysteretic model was originally designed for reinforced concrete beams. The differential description of the model, applied herein, is due to Minai and Suzuki <sup>6</sup>.

### 3. HYSTERETIC MODEL FOR MDOF SYSTEM

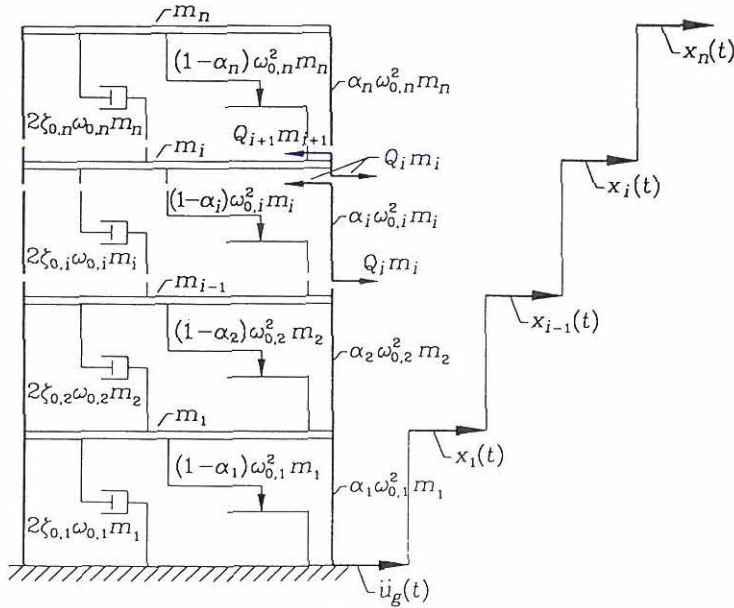


Figure 2. MDOF hysteretic oscillator model for the measuring points.

The horizontal displacement of the structure at a finite number of points is measured. The relative displacement between the  $i$ th and  $(i + 1)$ th measure points is designated  $x_i$ , and  $x_1$  signifies the displacement of the first measure point relative to the ground surface excited by the horizontal acceleration,  $\ddot{u}_g$ , see figure 2. For simplicity, the time dependence of  $x, \ddot{u}_g$  etc. is not explicitly shown in the following notation used for

the MDOF system. With reference to the shear model shown in figure 2, the relative displacement  $x_i$  between the  $i$ th and  $(i + 1)$ th measure point is assumed to cause a shear force of magnitude  $Q_i m_i$  where  $m_i$  is the storey mass. The equations of motion in terms of the relative displacements can be written as,

$$\left. \begin{aligned} \ddot{x}_1 &= \mu_2 Q_2 - Q_1 - \ddot{u}_g, & t > t_0 \\ \ddot{x}_i &= \mu_{i+1} Q_{i+1} - (\mu_i + 1) Q_i + Q_{i-1}, & t > t_0, \quad i = 2, 3, \dots, n-1 \\ \ddot{x}_n &= -(\mu_n + 1) Q_n + Q_{n-1}, & t > t_0 \\ x_i(t_0) &= \dot{x}_i(t_0) = 0, & i = 1, 2, \dots, n \end{aligned} \right\} \quad (9)$$

$$Q_i = 2\zeta_{0,i}\omega_{0,i}\dot{x}_i + \omega_{0,i}^2(\alpha_i x_i + (1 - \alpha_i)z_i), \quad i = 1, 2, \dots, n \quad (10)$$

$$\dot{z}_i = k(\dot{x}_i, z_i, D_i; z_{0,i})\dot{x}_i, \quad t > t_0, \quad z_i(t_0) = 0, \quad i = 1, 2, \dots, n \quad (11)$$

$$\dot{D}_i = g(\dot{x}_i, z_i; z_{0,i})\dot{x}_i, \quad t > t_0, \quad D_i(t_0) = D_{i,0}, \quad i = 1, 2, \dots, n \quad (12)$$

$$\alpha_i = \left( \frac{2z_{0,i}}{2z_{0,i} + D_i} \right)^{n_{0,i}}, \quad i = 1, 2, \dots, n \quad (13)$$

$$\mu_i = \frac{m_i}{m_{i-1}}, \quad i = 2, 3, \dots, n \quad (14)$$

In (9),  $n$ th measure point is assumed to be located at the top storey.  $k(\dot{x}_i, z_i, D_i; z_{0,i})$  and  $g(\dot{x}_i, z_i; z_{0,i})$  are given by (5) and (6).  $2\zeta_{0,i}\omega_{0,i}m_i$  and  $\omega_{0,i}^2m_i$  are respectively the damping coefficients and initial elastic spring stiffnesses between the storeys. Hence,  $\omega_{0,i}$  and  $\zeta_{0,i}$   $i = 1, 2, \dots, n$  are merely parameters to specify the linear parts of the shear forces and should not be confused with the natural frequencies and damping ratios of the structure. These parameters along with  $\mu_i$   $i = 2, 3, \dots, n$  must be identified so that the elastic model of (9) and (10) with  $\alpha_i = 1$  provides the same undamped circular eigenfrequencies  $\omega_i$ , damping ratios  $\zeta_i$  and modal participation factors  $\beta_i$  of the undamaged structure, as calculated or measured by non-destructive testing. Notice that the indicated discrete linear system has  $3n - 1$  free parameters,  $\omega_{0,i}$ ,  $\zeta_{0,i}$  and  $\mu_i$ , to fit the  $3n$  parameters,  $\omega_i$ ,  $\zeta_i$  and  $\beta_i$ , obtained from the primary linear system identification of the structure. Here, it is assumed that only the lowest  $n$  modes of the primary linear structure have been identified. There is an indeterminateness in the secondary system identification. This means that conditions can only be met at the lowest  $n - 1$  modes.

The hysteretic parameters  $z_{0,i}$  and  $n_{0,i}$  are sequentially updated during the damage process after each severe earthquake by system identification.



#### 4. GLOBAL DAMAGE INDICATORS : MAXIMUM SOFTENING INDEX

Consider the SDOF model. The instantaneous softening,  $\delta(t)$ , of the structure is defined as, Çakmak et al. <sup>1,2,3</sup>.

$$\delta(t) = 1 - \frac{T_0}{T(t)} \quad (15)$$

where  $T_0$  is the first period of the linear structure and  $T(t)$  is the first period of the equivalent linear structure with slowly varying stiffness characteristics during an earthquake excitation.  $T_0$  is assumed to be known from previous structural analysis or non-destructive experimentation of the structure and  $T(t)$  is estimated from the excitation and displacement response time series of the experienced earthquake.

The maximum softening damage indicator,  $\delta_M$  is the maximum of  $\delta(t)$  during the seismic excitation.

$$\delta_M = \max \delta(t) \quad (16)$$

In the hysteretic model, the instantaneous slope of the hysteretic curve defines the varying instantaneous period of the equivalent linear structure. For Clough-Johnston model, the instantaneous slope is  $A(t)$  for loading branches, 1 for unloading branches and 0 when yielding occurs. Therefore, instead of instantaneous softening, an average softening value is defined using the average slope,  $\bar{m}$ , of the hysteresis loop, the slope of the line through extreme points.

$$\bar{m} = \frac{2z_0}{2z_0 + D(t)} \quad (17)$$

The loop-averaged softening  $S(t)$  is

$$S(t) = 1 - \sqrt{\frac{2z_0}{2z_0 + D(t)} (1 - \alpha(D(t))) + \alpha(D(t))} \quad (18)$$

where  $\alpha(D(t))$  is given by (4). As seen from (18),  $S(t)$  is non-decreasing during a seismic event and fully correlated to  $D(t)$ .  $S(t)$  can then only measure the effect of the plastic deformations on the period of the structure since  $z_0$  is considered to be non-degrading during a seismic event in this study.

Correspondingly, local softening can be defined for each measure point in the MDOF system.

$$S_i(t) = 1 - \sqrt{\frac{2z_{0,i}}{2z_{0,i} + D_i} (1 - \alpha_i) + \alpha_i} \quad , i = 1, 2, \dots, n \quad (19)$$



where  $\alpha_i$  is given by (13).  $S_i(t)$  is a local damage indicator displaying the damaging effects of the local plastic deformations.

## 5. SYSTEM IDENTIFICATION

The proposed global damage indicator and the hysteretic model for the SDOF system are defined by six parameters, namely,  $T(t)$ ,  $\zeta_0$ ,  $\omega_0$ ,  $\beta_0$ ,  $z_0$ ,  $n_0$ .  $T(t)$  is estimated from the excitation and displacement response time series of the experienced earthquake using an ARMA model, e.g. <sup>5</sup>, suited to the displacement response process and the estimated ARMA coefficients are mapped to determine the first period and the instantaneous softening,  $S(t)^{ARMA}$ , of the corresponding dynamic system. The next three parameters are assumed to be known before the arrival of the first earthquake, via linear structural analysis or previous non-destructive experimentation. The hysteresis parameters  $z_0$  and  $n_0$  are estimated from an iterative system identification procedure where a weighted error criterion for the  $j$ th iteration,  $F^j(x(t), S(t); \hat{z}_0^j, \hat{n}_0^j)$ , defined from the instantaneous softening and the displacement response time series is employed.

$$F^j(x(t), S(t); \hat{z}_0^j, \hat{n}_0^j) = w_1 \sum_k (\hat{x}_k^j - x_k)^2 + w_2 \sum_l (\hat{S}_l^j - S_l^{ARMA})^2 \quad (20)$$

where  $x_k = x(t = t_k)$  etc.  $x_k$  are the measured displacements.  $\hat{x}_k^j$  and  $\hat{S}_l^j$  are model predictions using estimated hysteretic parameters,  $\hat{z}_0^j$ ,  $\hat{n}_0^j$ . The summation over the index  $k$  is performed for the time interval where large oscillations occur, and, the summation over the index  $l$  is performed for the time interval where  $S^{ARMA}(t)$  more or less stabilizes. This guarantees more weights given to large oscillations and large damage levels. Additionally,  $w_1$  and  $w_2$  are chosen such that the displacement and instantaneous softening contributions in the error are approximately equal, i.e.

$$\frac{w_1 \sum_k (\hat{x}_k^j - x_k)^2}{w_2 \sum_l (\hat{S}_l^j - S_l^{ARMA})^2} \simeq 1 \quad (21)$$

The new estimates for the hysteretic parameters are then obtained using the steepest descent method, e.g. <sup>5</sup>.

$$z_0^{j+1} = z_0^j - \frac{\partial F^j}{\partial z_0} \epsilon_{z_0} \quad , \quad n_0^{j+1} = n_0^j - \frac{\partial F^j}{\partial n_0} \epsilon_{n_0} \quad (22)$$

the gradients  $\frac{\partial F^j}{\partial z_0}$  and  $\frac{\partial F^j}{\partial n_0}$  are computed numerically.  $\epsilon_{z_0}$  and  $\epsilon_{n_0}$  are step parameters calibrated from the numerical values of the gradients.

By these system identification procedures,  $T(t)$  and hysteresis parameters  $z_0$  and  $n_0$  have been estimated using the observable measures; the excitation and displacement response time series only.

For MDOF systems, similar system identification methods can be introduced.

## 6. PREDICTION OF DAMAGE AND RELIABILITY

It will be assumed that the excitations from different earthquakes are mutually stochastically independent. Then, the memory of the previous earthquakes is carried over to the state vector  $\mathbf{Z}^T(t) = [\mathbf{x}(t), \dot{\mathbf{x}}(t), \mathbf{z}(t), \mathbf{D}(t)]$  where  $\mathbf{x}^T(t) = [x_1(t), \dots, x_n(t)]$  etc. Since, it has been assumed above that the structural system returns to the equilibrium state after the previous earthquake corresponding to the initial values  $\mathbf{x}(t_0) = \dot{\mathbf{x}}(t_0) = \mathbf{z}(t_0) = \mathbf{0}$ , the memory of the previous earthquakes is then carried totally by the initial values  $\mathbf{D}(t_0) = \mathbf{D}_0$ . With these conditions on the initial values of the other state variables, the initial values of the damage vector  $\mathbf{D}(t)$  in a sequence forms a Markov chain to the extent that the present mathematical model is an adequate representation of the RC-structure. Since the softening  $S(t)$  and  $\mathbf{S}(t)$  of the SDOF and MDOF systems only depend on the initial values  $D_0$  and  $\mathbf{D}_0$  of the damage process and the present earthquake  $\ddot{u}_g(t)$ , these quantities form a Markov chain. The consequences of these statements are that both the damage process and the reliability as measured using the maximum softening value in future earthquakes, can be predicted by the present model, if only the damage process,  $D(t)$  and  $\mathbf{D}(t)$  are updated after each seismic event, i.e. their terminal values in the previous earthquakes are calculated via updated hysteretic model parameters,  $z_0$ ,  $n_0$  and  $\mathbf{z}_0$ ,  $\mathbf{n}_0$ .

The reliability of the structure subject to future earthquakes can be estimated using independent Monte Carlo simulations. This requires a suitable earthquake model for the generation of the mutually independent earthquake excitations. Running the hysteretic model subject to these excitations generates a sample set for the numerical values of maximum softening. These samples can further be used in evaluating the reliability of the structure based on the definition of failure event.

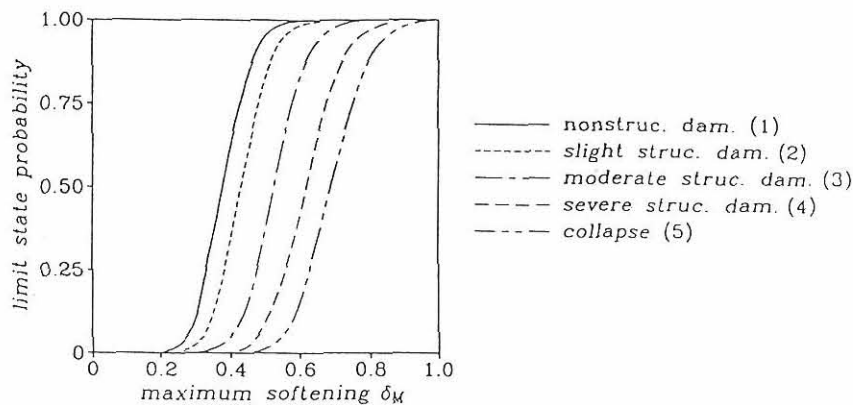


Fig.3: Distribution of observed limit state values of one-dimensional maximum softening <sup>3</sup>.

In figure 3, the distribution function of observed values of the one-dimensional maximum softening, obtained from analysis of shake table experiments with reinforced concrete frames (Healey and Sozen (1978), Cecen (1979)), is shown. The limit state definition varied from insignificant damage to total collapse. In all cases a relatively small coefficient of variation was observed for the damage indicator. From this analysis it is



concluded that safe states of the structure can be separated from unsafe states with sufficient reliability, by determining whether or not the maximum softening  $\delta_M$  exceeds a critical value  $\delta_0$ . The failure event is then defined by  $\{\delta_M \geq \delta_0\}$ . For MDOF systems  $n$ -dimensional failure surfaces can be defined similarly<sup>9</sup>.

## 7. EXAMPLE

In order to demonstrate the ability of the proposed hysteretic SDOF model to fit and predict actual seismic response of RC structures, experimentally recorded results on a 1:10 scaled planar 10 storey 3 bay reinforced concrete frame, shown in figure 4, tested at University of Illinois at Urbana Champaign<sup>5</sup>, are used.

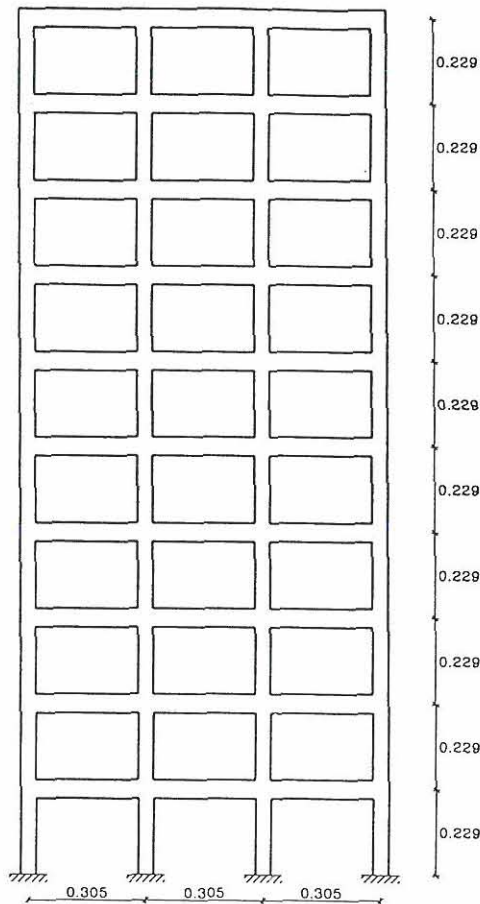


Figure 4. 10 storey 3 bay reinforced concrete frame.

The test structure consisted of two parallel frames working in parallel with ten uniformly distributed storey weights, attached in between. The beams and columns are symmetrically reinforced so that yield limits are the same in compression and tension, see<sup>5</sup> for more information about the geometrical and structural details of the structure.

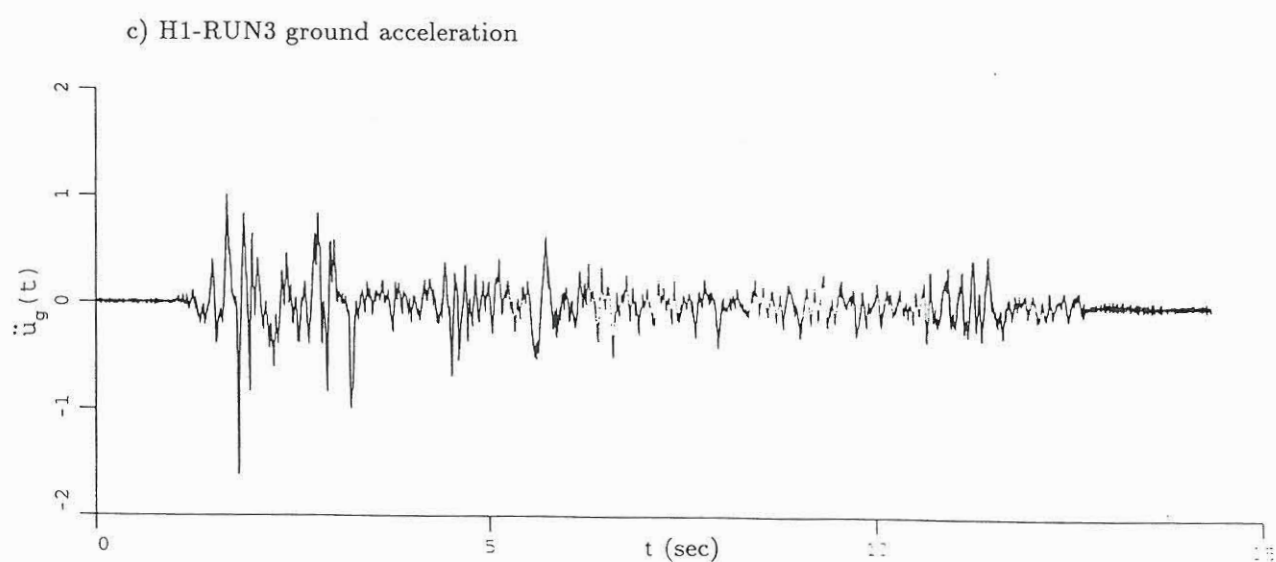
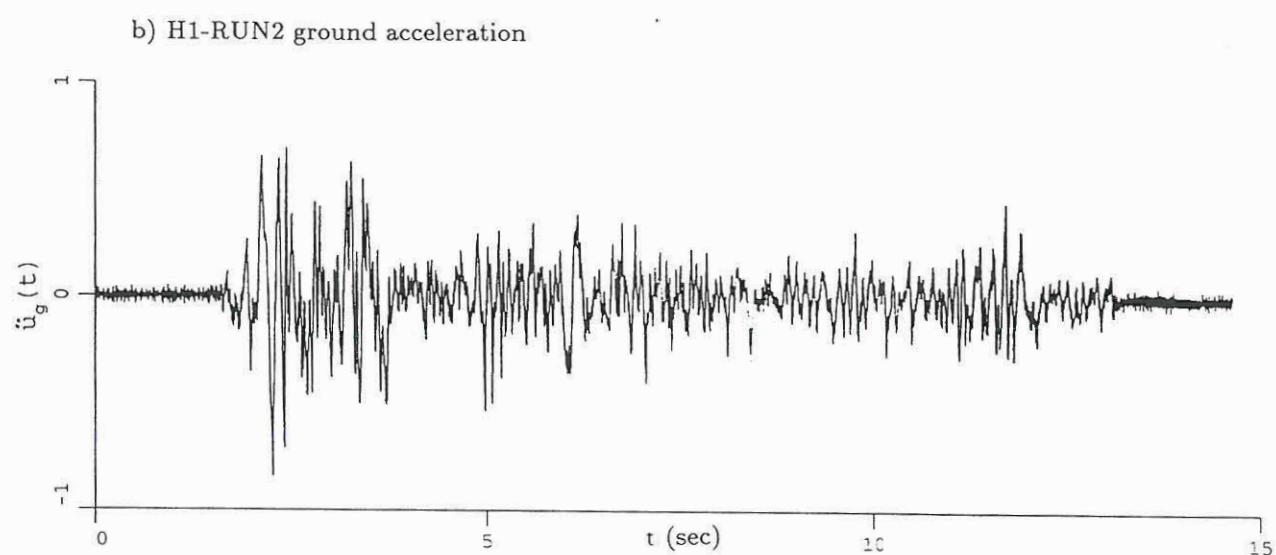
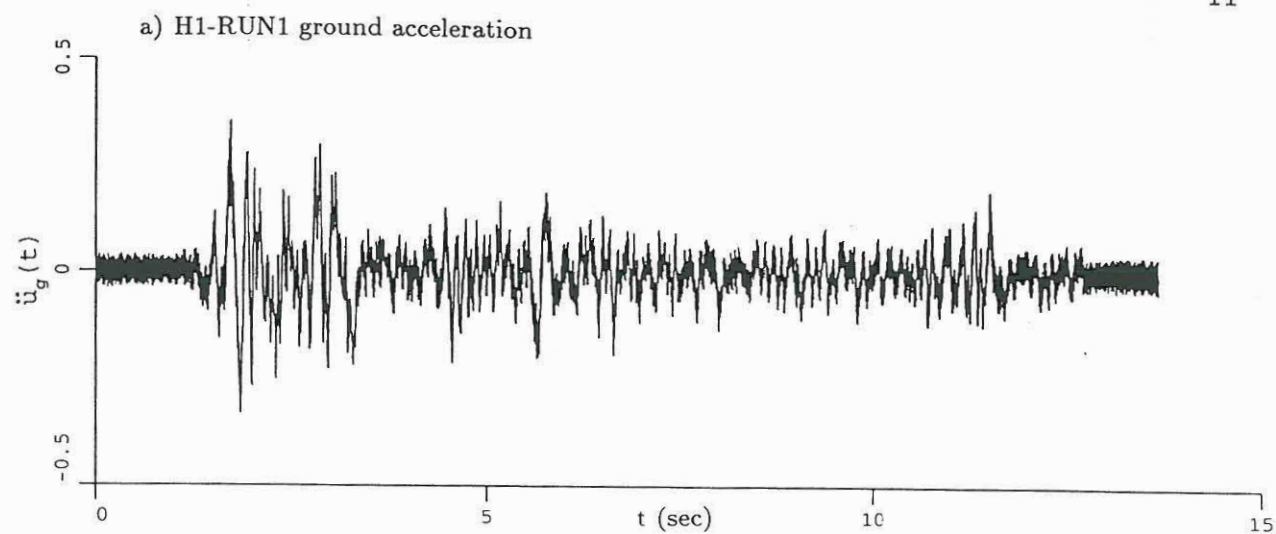


Figure 5. Ground accelerations normalized with respect to  $g = 9810 \text{ mm/sec}^2$  versus time,  $t$ , in seconds.



The eigenfrequency, damping ratio and modal participation factor for the first mode of the undamaged structure are  $\omega_0 = 6\pi$ ,  $\zeta_0 = 0.035$  and  $\beta_0 = 1.32$ . The first eigenvector is chosen such that displacement of the top storey is  $1.32x(t)$  where  $x(t)$  is the first modal coordinate.

This structure is excited by 3 consecutive horizontal acceleration processes at the ground surface which are simulated models of the El-Centro earthquake in 1940. These tests are called H1-RUN1, H1-RUN2 and H1-RUN3 in <sup>5</sup>, so they are in this paper. The horizontal ground surface accelerations of these runs are given in figure 5 where  $\ddot{u}_g(t)$  is normalized by the gravitational constant  $g = 9810 \text{ mm/sec}^2$ .

$T(t)$  is estimated from the excitation and displacement response time series of the experienced earthquake using an ARMA model, e.g. <sup>5</sup>, suited for the displacement response process. The time window size is chosen as 2.4 seconds and an ARMA model is fit for each 2.4 second time window. The estimate is located at the centre of the window and the estimates are smoothed.  $S^{ARMA}(t)$  is then obtained using (15), see figure 8.

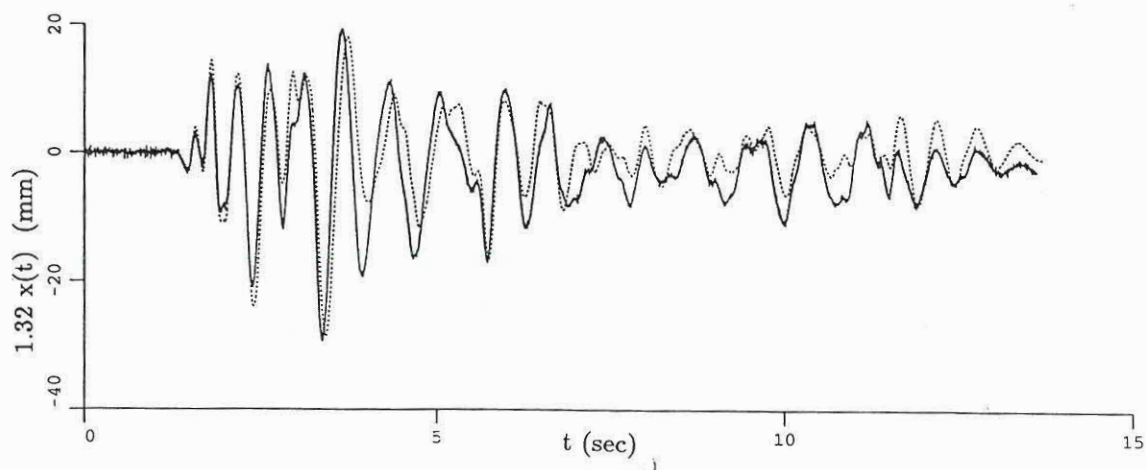
$z_0$  and  $n_0$  are estimated using the predescribed steepest descent method. The identified numerical values for  $z_0$  and  $n_0$  using the corresponding runs are listed in table 1.

RUN	$z_0$ (cm)	$n_0$
RUN1	2.68	0.83
RUN2	3.01	0.77
RUN3	3.14	0.73

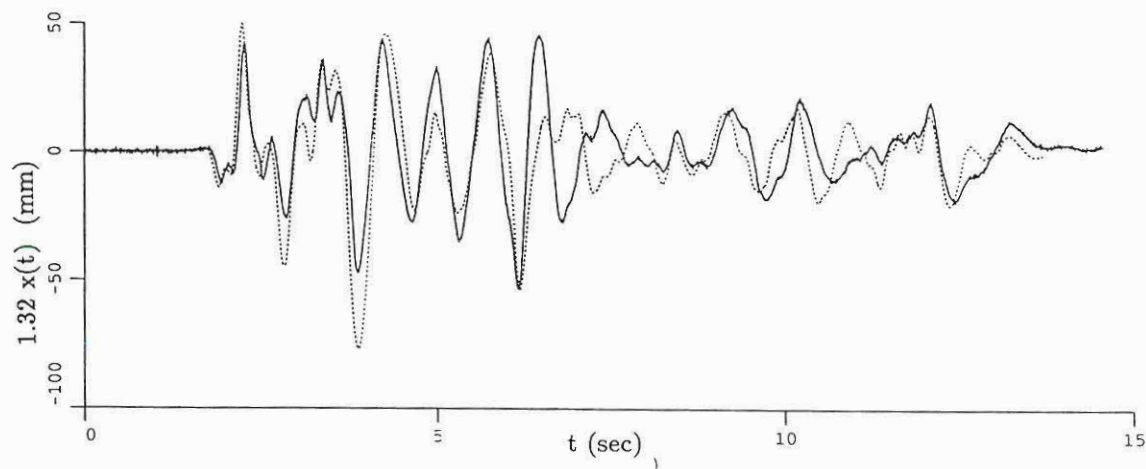
Table 1. Estimated hysteretic parameters

The performance of the suited hysteretic models for each run using the values listed in table 1 is shown in figures 6 and 7 for the displacement of the top storey,  $1.32x(t)$ , and softening,  $S(t)$ , respectively. The prediction performance of the suited hysteretic model for RUN1 in the future earthquakes of tests RUN2 and RUN3 is shown in figures 8 and 9. Model predictions for  $\hat{S}(t)$  are very good. The prediction performance of the suited hysteretic model for RUN2 in the future earthquake of test RUN3 is shown in figures 10 and 11. Comparison of figures 8 and 10, and 8 and 11 show that the updated model predicts better.

a) H1-RUN1 displacement of the top storey



b) H1-RUN2 displacement of the top storey



c) H1-RUN3 displacement of the top storey

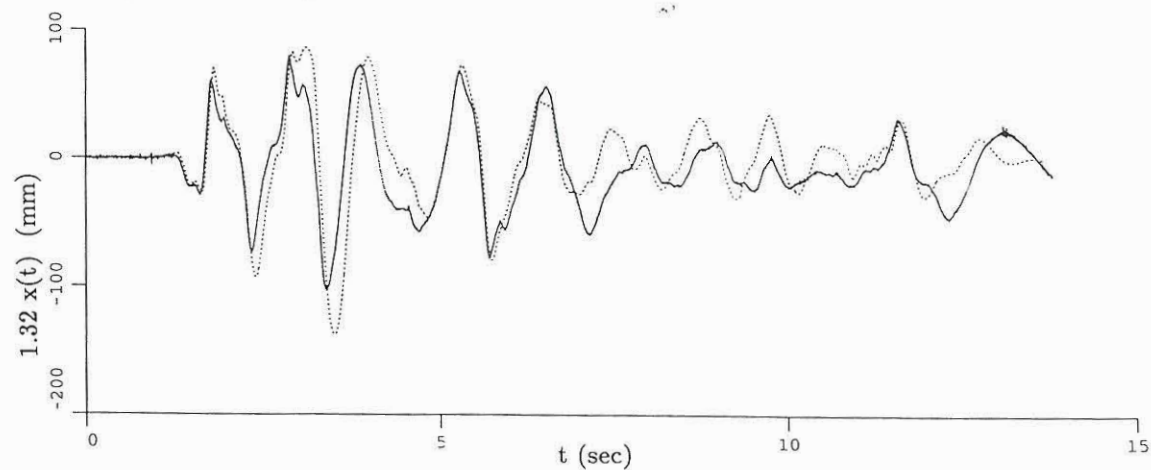


Figure 6. The performance of the suited hysteretic models for each RUN. Displacement of the top storey,  $1.32x(t)$  and  $1.32\hat{x}(t)$  in mm, versus time,  $t$ , in seconds.



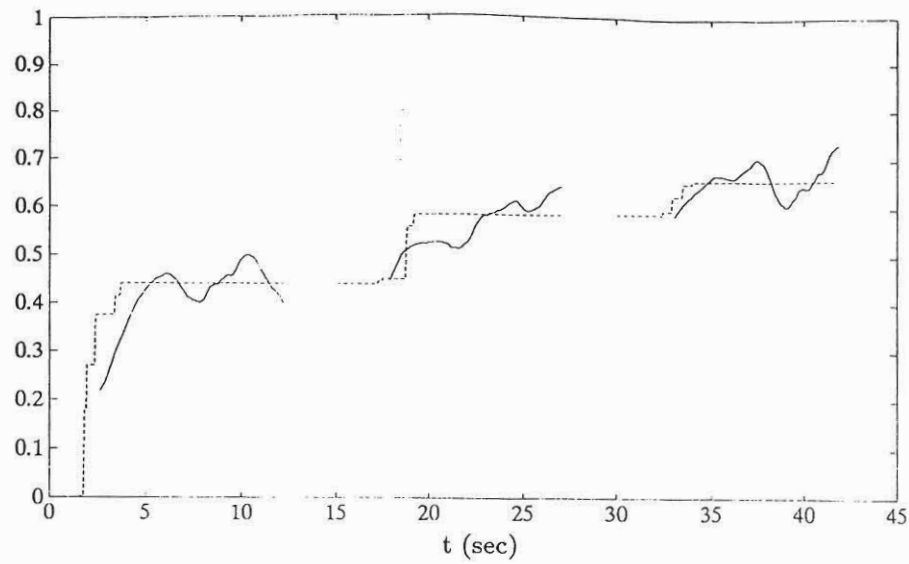


Figure 7. The performance of the suited hysteretic models for each RUN. Softening,  $S^{ARMA}(t)$  and  $\hat{S}(t)$ , versus time,  $t$ , in seconds.

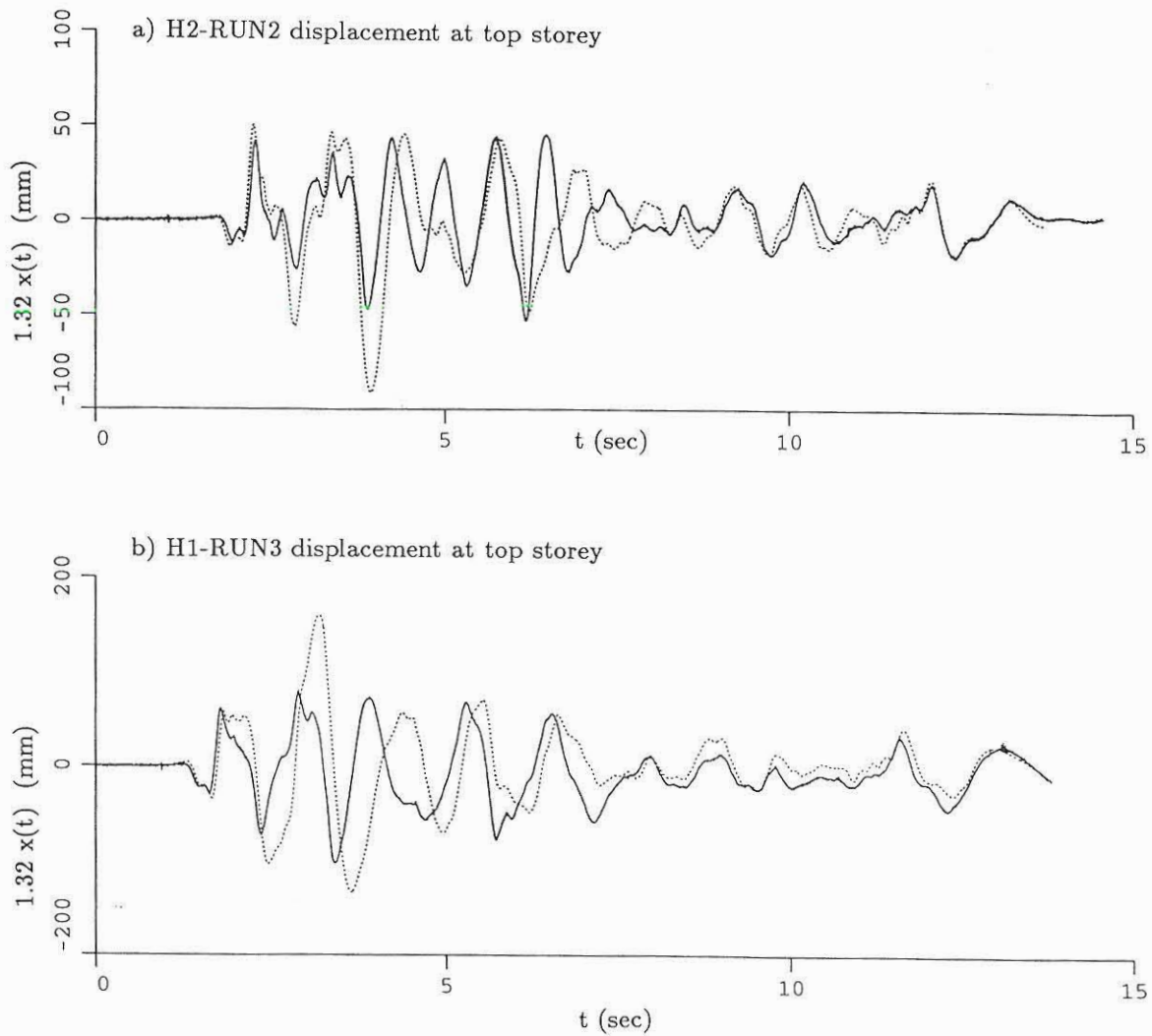


Figure 8. The prediction performance of the suited hysteretic model for RUN1 in RUN2 and RUN3 cases. Displacement of the top storey,  $1.32x(t)$  and  $1.32\hat{x}(t)$  in mm, versus time,  $t$ , in seconds.

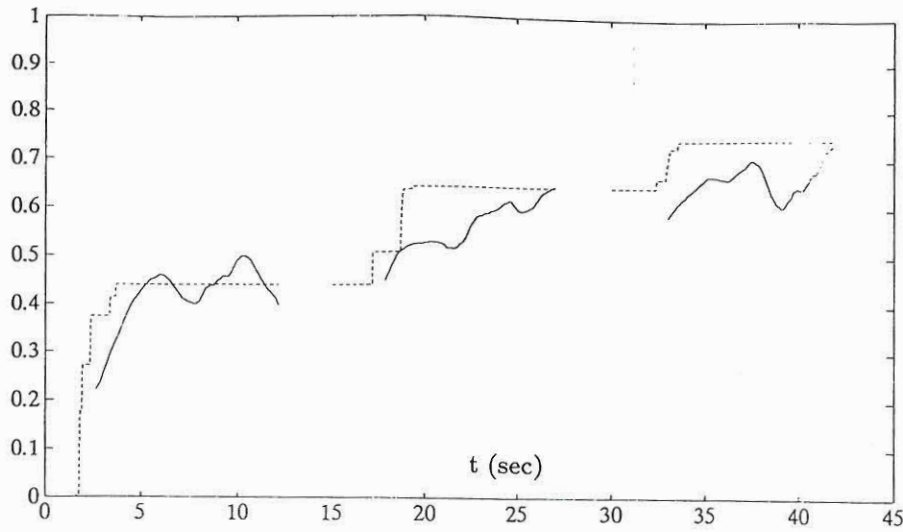


Figure 9. The prediction performance of the suited hysteretic model for RUN1 in RUN2 and RUN3 cases. Softening,  $S^{ARMA}(t)$  and  $\hat{S}(t)$ , versus time,  $t$ , in seconds.

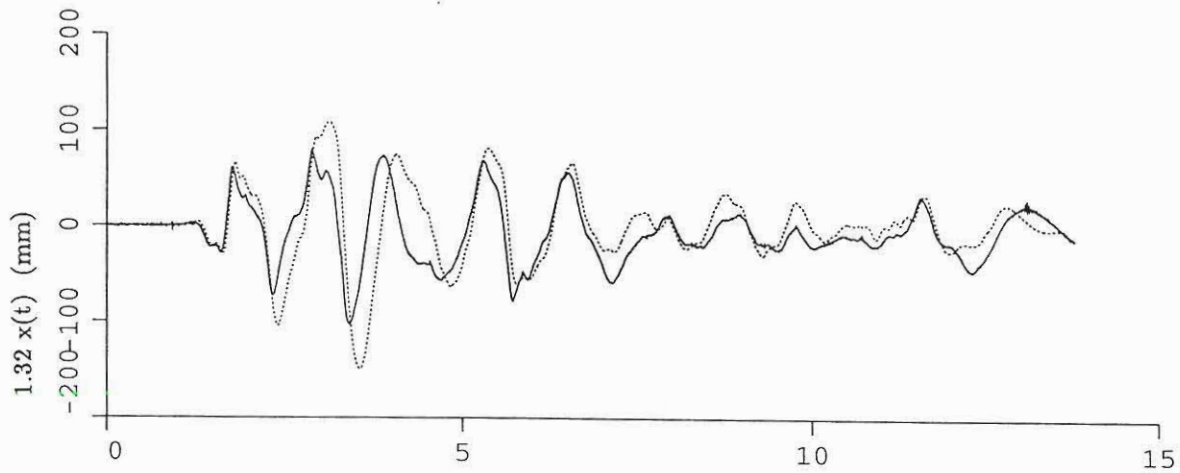


Figure 10. The prediction performance of the suited hysteretic model for RUN2 in RUN3 case. Displacement of the top storey,  $1.32x(t)$  and  $1.32\hat{x}(t)$  in mm, versus time,  $t$ , in seconds.

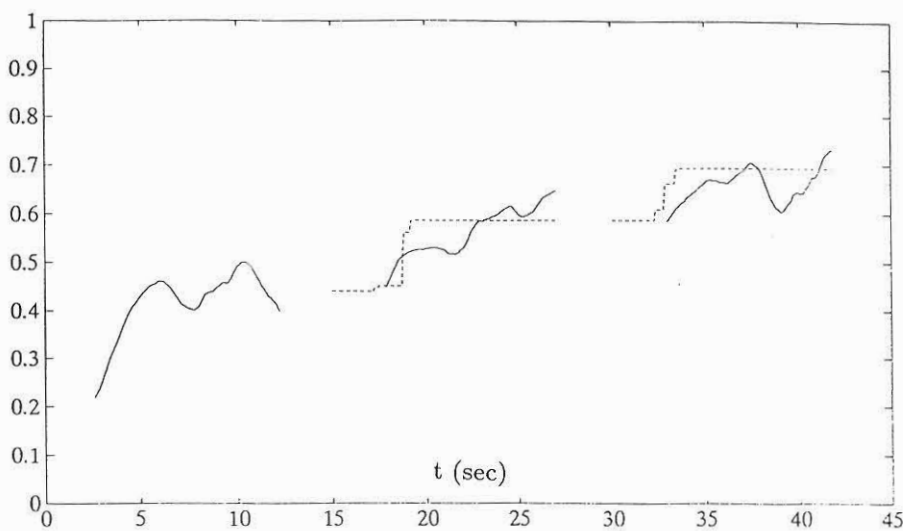


Figure 11. The prediction performance of the suited hysteretic model for RUN2 in RUN3 case. Softening,  $S^{ARMA}(t)$  and  $\hat{S}(t)$ , versus time,  $t$ , in seconds.



Figure 12 shows the results obtained in earlier works by MDOF modelling of this structure using non-linear beam theories. The computer programs used in these are SARCF-II<sup>10</sup> and SARCOF<sup>7</sup>. A comparison of figure 12 with figures 6, 8 and 10 show that the proposed model fits better than the other two approaches, primarily because of the degrading model of the  $\alpha(D(t))$  parameter.

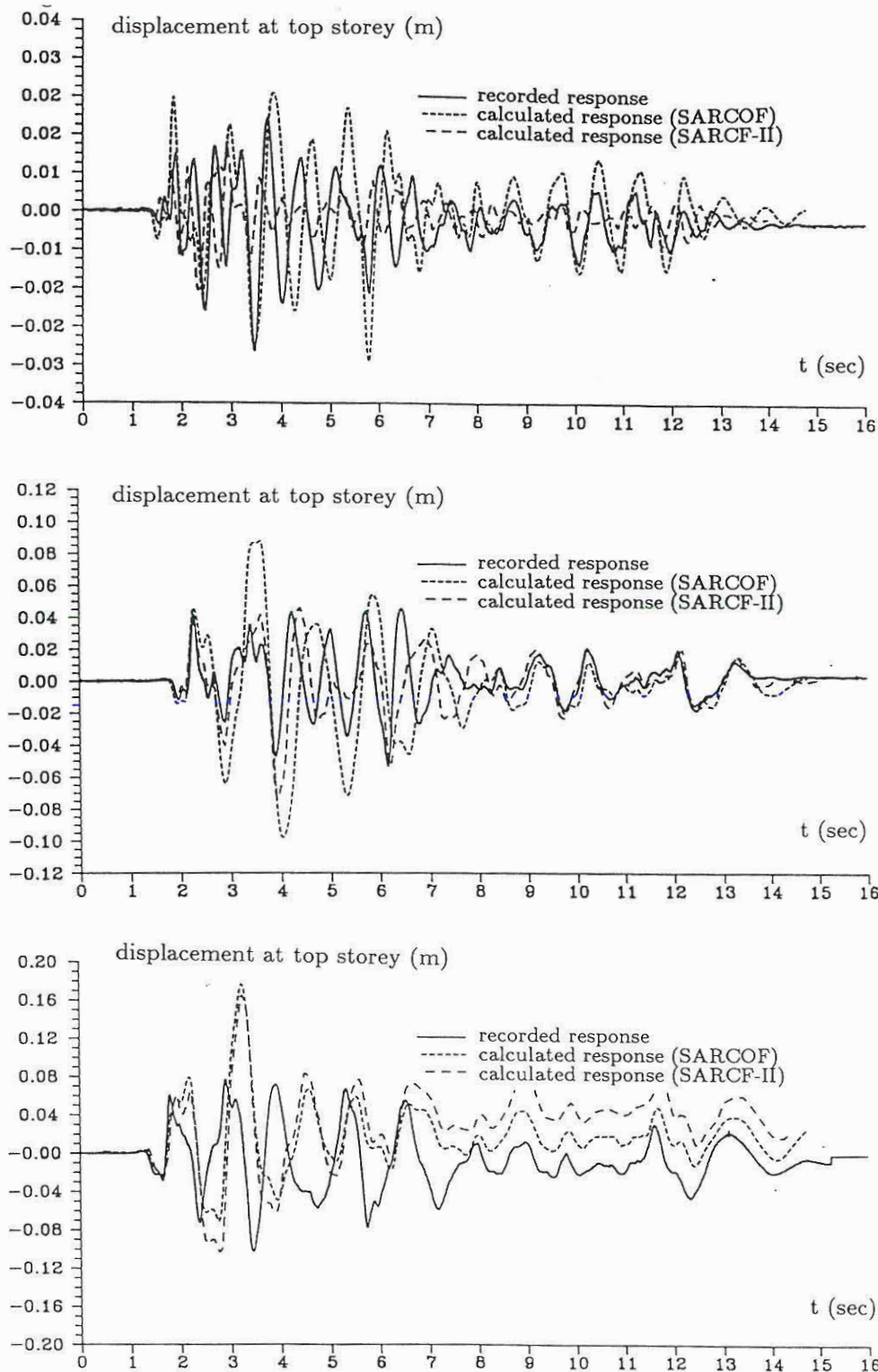


Figure 12. The performance of earlier models using SARCF-II and SARCOF for RUN1, RUN2 and RUN3. Displacement of the top storey in mm, versus time,  $t$ , in seconds.

## 8. CONCLUSIONS

A robust method is developed for the prediction of global and localized damage and the future reliability estimation of partly damaged reinforced concrete (RC) structures under seismic excitation. A global maximum softening damage indicator is considered based on the variation of the eigenfrequency of the first mode due to the stiffness and strength deterioration of the structure. The hysteresis of the first mode is modelled by a Clough and Johnston hysteretic oscillator <sup>1</sup>, with degrading elastic fraction of the restoring force. The linear parameters of the model are assumed to be known, measured before the arrival of the first earthquake from non-destructive vibration tests or via structural analysis. The previous excitation and displacement response time series is employed for the identification of the instantaneous softening using an ARMA model. The hysteresis parameters are updated after each earthquake. The proposed model is next generalized for the MDOF system. Using the adapted models for the structure and the global damage state, the global damage in a future earthquake can then be estimated if a suitable earthquake model is applied. The performance of the model is illustrated by RC frames which were tested by Sözen and his associates <sup>2,4</sup> and the predicted results in terms of instantaneous softening and top storey displacement are in very good agreement with the ones recorded during the experiments.

## 9. ACKNOWLEDGEMENTS

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